American University of Beirut

MATH 202

Differential Equations

Spring 2007

Final Exam

Name: .... ID #: ....

circle your section please: **13 (T 11)** **14 (T 12:30)** **15 (T 2)** **16 (T 9:30)**

1. *(10 points)* Find the general solution of

$$y^{(4)} - y = e^x$$

2. *(15 points)* Use a power series centered at the point $x = 0$ to find the general solution of:

$$(1 + x^3)y'' - 6xy = 0$$

(simplify the recurrence as much as possible before solving for $c_a, c_1, \ldots$)

3. *(15 points)* Find a) $\mathcal{L}\left\{(3t+1)\mathcal{U}(t-1)\right\}$ b) $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2(s-1)}\right\}$ c) $\mathcal{L}\left\{t\displaystyle\int_0^t \tau e^{-\tau}\,d\tau\right\}$

4. a. *(6 points)* Use the substitution $x = e^t$ to solve the following differential equation in terms of Bessel functions:

$$\frac{d^2y}{dt^2} + \left(e^{2t} - \frac{1}{4}\right)y = 0$$

b. *(4 points)* express your answer in terms of the elementary functions

c. *(10 points)* Show that $y = \sqrt{x}J_\nu(x)$ is a solution of the differential equation

$$x^2y'' + \left(x^2 - \nu^2 + \frac{1}{4}\right)y = 0$$

5. *(15 points)* Use the Laplace transform to solve the following DE:

$$y' + y = f(t)\ , \quad y(0) = 5$$

where $f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 3\cos t & t \ge \pi \end{cases}$

6. *(15 points)* Use a generalized (i.e. Frobenius) power series of the form $y = \displaystyle\sum_{n=0}^{+\infty} c_n x^{n+r}$ to find **ONE** solution of

$$3x^2y'' + 2xy' - xy = 0$$

corresponding to the **LARGER** indicial roots

7. *(10 points)* Find the general solution of the given system

$$\begin{cases} \dfrac{dx}{dt} = x + 2y \\ \dfrac{dy}{dt} = 4x + 3y \end{cases}$$